**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

Ans: B

the serving work will begin after 10 min of drop off so 45+10 which will now take more than the usual time so new mew is 55 minutes and the probability that it will take more than 1 hour to complete.

mew = 55, std = 8

from scipy import stats

q1 = 1-stats.norm.cdf(60, loc = mew, scale = std)

q1 = 0.2659 The probability that the service manager cannot meet his commitment is 0.2659

or

Since the service manager plans that servicing begins after 10 mins, hence the available time for servicing = 60 – 10 = 50 minutes

By using distribution function

1-stats.norm.cdf(50,45,8) = 0.2676

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.
3. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Ans:

1. First find out the probability for employees older than 44:

X = 44, mean =38, std = 6

1-stats.norm.cdf(44, 38, 6) = 1-0.8413

=0.1587

Now find the probability of employees between 38 and 44:

stats.norm.cdf(44, 38, 6) – stats.norm.cdf(38,38,6)

=0.8413 – 0.5

=0.3413

False: Because the probability for employees at the processing center are more between

38 and 44 than older than 44.

1. Lets calculate the probability of employees age under 30:

X = 30, mean = 38, std=6

Stats.norm.cdf(30,38,6) = 0.0912

So the total number of employees age under 30 is 0.0912 \* 400 =36.48

So we can say that the statement is True.

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

Ans:

2X1 = *N*(2μ, 4σ2)

X1 + X2 = *N*(2μ, 2σ2)

[2X1 – (X1 + X2)] = N(0, 2σ2)

* The mean of 2X1 and X1 + X2 are same = 2 μ
* The variance of 2X1 is twice as the variance of X1 + X2

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

Ans: D

The two values of a and b, symmetric about the mean, are such that the probability of the random variable taking a value between them is 0.99:

from scipy import stats

stats.norm.interval(0.99,mu=100,loc=20) = 48.5,151.5

Or

By checking the options separately,

Eg: stats.norm.cdf(151.5,100,20)-stats.norm.cdf(48.5,100,20) = 0.99

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

In Coding File

1. Specify the 5th percentile of profit (in Rupees) for the company

Ans:

5th percentile profit

stats.norm.ppf(0.05) = -1.6448536269514729

In normal distribution, Z = (x- μ)/ σ

X = σ Z + μ

For the 5th percentile (stats.norm.ppf(0.05)) , z= -1.64, μ = 540, σ = 225

X = 540 + (-1.64\*225)

= 171 million rupees

1. Which of the two divisions has a larger probability of making a loss in a given year?

Ans:

For making loss, X<0

print(“The probability of profit\_1 for loss”,stats.norm.cdf(0,5,3))

print(“The probability of profit\_2 for loss”,stats.norm.cdf(0,7,4))

The probability of Profit\_1 for loss 0.0477

The probability of Profit\_2 for loss 0.0400

Therefore the **2nd divisions** are going to make more loss as there **probability is less**